

Paper Reference(s)

6666/01

Edexcel GCE

Core Mathematics C4

Silver Level S5

Time: 1 hour 30 minutes

Materials required for examination
papers

Mathematical Formulae (Green)

Items included with question

Nil

Candidates may use any calculator allowed by the regulations of the Joint Council for Qualifications. Calculators must not have the facility for symbolic algebra manipulation, differentiation and integration, or have retrievable mathematical formulas stored in them.

Instructions to Candidates

Write the name of the examining body (Edexcel), your centre number, candidate number, the unit title (Core Mathematics C4), the paper reference (6666), your surname, initials and signature.

Information for Candidates

A booklet 'Mathematical Formulae and Statistical Tables' is provided.

Full marks may be obtained for answers to ALL questions.

There are 7 questions in this question paper. The total mark for this paper is 75.

Advice to Candidates

You must ensure that your answers to parts of questions are clearly labelled.

You must show sufficient working to make your methods clear to the Examiner. Answers without working may gain no credit.

Suggested grade boundaries for this paper:

A*	A	B	C	D	E
68	59	50	46	39	31

1. The curve C has the equation $2x + 3y^2 + 3x^2y = 4x^2$.

The point P on the curve has coordinates $(-1, 1)$.

(a) Find the gradient of the curve at P .

(5)

(b) Hence find the equation of the normal to C at P , giving your answer in the form $ax + by + c = 0$, where a , b and c are integers.

(3)

January 2012

2.

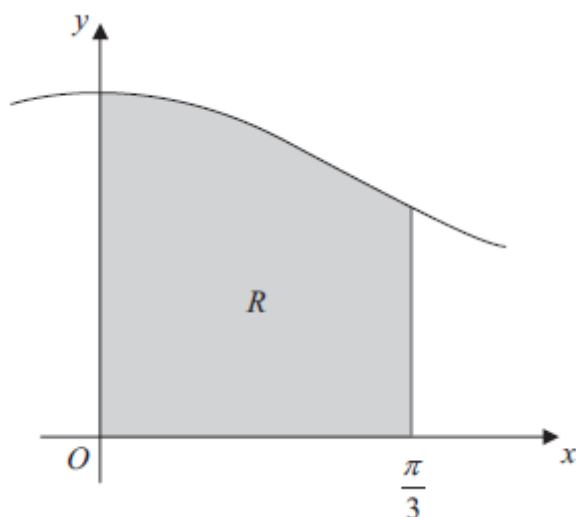


Figure 1

Figure 1 shows part of the curve with equation $y = \sqrt{0.75 + \cos^2 x}$. The finite region R , shown shaded in Figure 1, is bounded by the curve, the y -axis, the x -axis and the line with equation $x = \frac{\pi}{3}$.

(a) Copy and complete the table with values of y corresponding to $x = \frac{\pi}{6}$ and $x = \frac{\pi}{4}$.

x	0	$\frac{\pi}{12}$	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$
y	1.3229	1.2973			1

(2)

(b) Use the trapezium rule

(i) with the values of y at $x = 0$, $x = \frac{\pi}{6}$ and $x = \frac{\pi}{3}$ to find an estimate of the area of R .

Give your answer to 3 decimal places.

(ii) with the values of y at $x = 0$, $x = \frac{\pi}{12}$, $x = \frac{\pi}{6}$, $x = \frac{\pi}{4}$ and $x = \frac{\pi}{3}$ to find a further

estimate of the area of R . Give your answer to 3 decimal places.

(6)

June 2010

3.

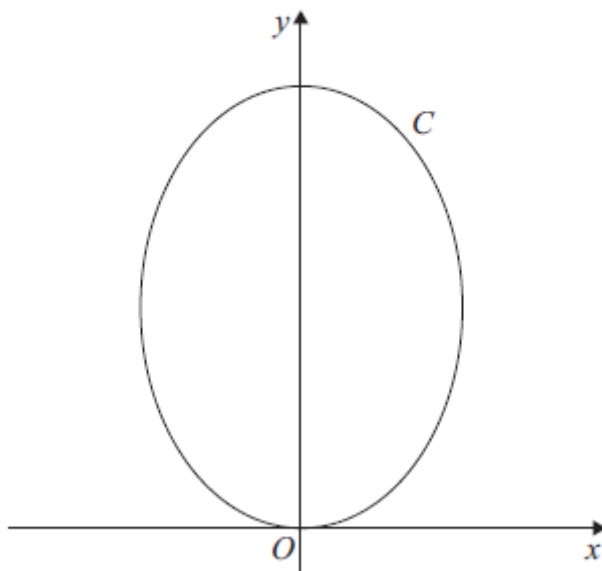


Figure 2

Figure 2 shows a sketch of the curve C with parametric equations

$$x = \sqrt{3} \sin 2t, \quad y = 4 \cos^2 t, \quad 0 \leq t \leq \pi.$$

(a) Show that $\frac{dy}{dx} = k\sqrt{3} \tan 2t$, where k is a constant to be determined.

(5)

(b) Find an equation of the tangent to C at the point where $t = \frac{\pi}{3}$.

Give your answer in the form $y = ax + b$, where a and b are constants.

(4)

(c) Find a cartesian equation of C .

(3)

June 2012

4. The line l_1 has vector equation

$$\mathbf{r} = \begin{pmatrix} -6 \\ 4 \\ -1 \end{pmatrix} + \lambda \begin{pmatrix} 4 \\ -1 \\ 3 \end{pmatrix}$$

and the line l_2 has vector equation

$$\mathbf{r} = \begin{pmatrix} -6 \\ 4 \\ -1 \end{pmatrix} + \mu \begin{pmatrix} 3 \\ -4 \\ 1 \end{pmatrix}$$

where λ and μ are parameters.

The lines l_1 and l_2 intersect at the point A and the acute angle between l_1 and l_2 is θ .

- (a) Write down the coordinates of A .

(1)

- (b) Find the value of $\cos \theta$.

(3)

The point X lies on l_1 where $\lambda = 4$.

- (c) Find the coordinates of X .

(1)

- (d) Find the vector \overrightarrow{AX} .

(2)

- (e) Hence, or otherwise, show that $|\overrightarrow{AX}| = 4\sqrt{26}$.

(2)

The point Y lies on l_2 . Given that the vector \overrightarrow{YX} is perpendicular to l_1 ,

- (f) find the length of AY , giving your answer to 3 significant figures.

(3)

January 2010

5. With respect to a fixed origin O , the lines l_1 and l_2 are given by the equations

$$l_1 : \mathbf{r} = (-9\mathbf{i} + 10\mathbf{k}) + \lambda(2\mathbf{i} + \mathbf{j} - \mathbf{k})$$

$$l_2 : \mathbf{r} = (3\mathbf{i} + \mathbf{j} + 17\mathbf{k}) + \mu(3\mathbf{i} - \mathbf{j} + 5\mathbf{k})$$

where λ and μ are scalar parameters.

- (a) Show that l_1 and l_2 meet and find the position vector of their point of intersection.

(6)

- (b) Show that l_1 and l_2 are perpendicular to each other.

(2)

The point A has position vector $5\mathbf{i} + 7\mathbf{j} + 3\mathbf{k}$.

- (c) Show that A lies on l_1 .

(1)

The point B is the image of A after reflection in the line l_2 .

- (d) Find the position vector of B .

(3)

June 2008

6. (a) Find $\int x \cos 2x \, dx$.

(4)

- (b) Hence, using the identity $\cos 2x = 2 \cos^2 x - 1$, deduce $\int x \cos^2 x \, dx$.

(3)

June 2007

7.

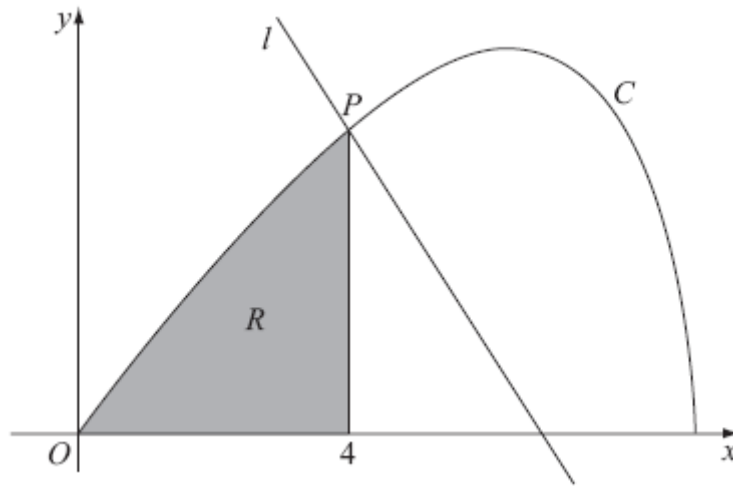


Figure 3

Figure 3 shows the curve C with parametric equations

$$x = 8 \cos t, \quad y = 4 \sin 2t, \quad 0 \leq t \leq \frac{\pi}{2}.$$

The point P lies on C and has coordinates $(4, 2\sqrt{3})$.

(a) Find the value of t at the point P .

(2)

The line l is a normal to C at P .

(b) Show that an equation for l is $y = -x\sqrt{3} + 6\sqrt{3}$.

(6)

The finite region R is enclosed by the curve C , the x -axis and the line $x = 4$, as shown shaded in Figure 3.

(c) Show that the area of R is given by the integral $\int_{\frac{\pi}{3}}^{\frac{\pi}{2}} 64 \sin^2 t \cos t \, dt$.

(4)

(d) Use this integral to find the area of R , giving your answer in the form $a + b\sqrt{3}$, where a and b are constants to be determined.

(4)

June 2008

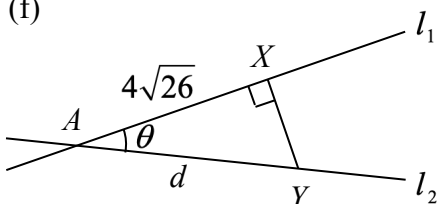
TOTAL FOR PAPER: 75 MARKS

END

Question Number	Scheme	Marks
1. (a)	$\left\{ \frac{\cancel{dy}}{\cancel{dx}} \times \right\} \underline{2 + 6y \frac{dy}{dx} + \left(\underline{6xy + 3x^2 \frac{dy}{dx}} \right) = \underline{8x}}$ $\left\{ \frac{dy}{dx} = \frac{8x - 2 - 6xy}{6y + 3x^2} \right\} \quad \text{not necessarily required.}$ <p>At $P(-1, 1)$, $m(\mathbf{T}) = \frac{dy}{dx} = \frac{8(-1) - 2 - 6(-1)(1)}{6(1) + 3(-1)^2} = -\frac{4}{9}$</p>	M1 <u>A1</u> <u>B1</u> dM1 A1 cs0 [5]
(b)	<p>So, $m(\mathbf{N}) = \frac{-1}{-\frac{4}{9}} \left\{ = \frac{9}{4} \right\}$</p> <p>N: $y - 1 = \frac{9}{4}(x + 1)$</p> <p>N: $9x - 4y + 13 = 0$</p>	M1 M1 A1 [3] (8 marks)

Question Number	Scheme	Marks
2.	<p>(a) $y\left(\frac{\pi}{6}\right) \approx 1.2247, y\left(\frac{\pi}{4}\right) = 1.1180$ accept awrt 4 d.p.</p> <p>(b)(i) $I \approx \left(\frac{\pi}{12}\right)(1.3229 + 2 \times 1.2247 + 1) \approx 1.249$ B1 for $\frac{\pi}{12}$ cao</p> <p>(ii) $I \approx \left(\frac{\pi}{24}\right)(1.3229 + 2 \times (1.2973 + 1.2247 + 1.1180) + 1) \approx 1.257$ B1 for $\frac{\pi}{24}$ cao</p>	<p>(2)</p> <p>B1 B1</p> <p>B1 M1</p> <p>A1</p> <p>B1 M1</p> <p>A1</p> <p>(6) [8]</p>

Question Number	Scheme	Marks
3. (a)	$\frac{dx}{dt} = 2\sqrt{3} \cos 2t$ $\frac{dy}{dt} = -8 \cos t \sin t$ $\frac{dy}{dx} = \frac{-8 \cos t \sin t}{2\sqrt{3} \cos 2t}$ $= -\frac{4 \sin 2t}{2\sqrt{3} \cos 2t}$ $\frac{dy}{dx} = -\frac{2}{3}\sqrt{3} \tan 2t \quad \left(k = -\frac{2}{3}\right)$	B1 M1 A1 M1 A1 (5)
(b)	When $t = \frac{\pi}{3}$ $x = \frac{3}{2}, y = 1$ can be implied $m = -\frac{2}{3}\sqrt{3} \tan\left(\frac{2\pi}{3}\right) \quad (= 2)$ $y - 1 = 2\left(x - \frac{3}{2}\right)$ $y = 2x - 2$	B1 M1 M1 A1 (4)
(c)	$x = \sqrt{3} \sin 2t = \sqrt{3} \times 2 \sin t \cos t$ $x^2 = 12 \sin^2 t \cos^2 t = 12(1 - \cos^2 t) \cos^2 t$ $x^2 = 12\left(1 - \frac{y}{4}\right) \frac{y}{4}$	M1 or equivalent M1 A1 (3)
		[12]

Question Number	Scheme	Marks
Q4	(a) $A: (-6, 4, -1)$ Accept vector forms	B1 (1)
	(b) $\begin{pmatrix} 4 \\ -1 \\ 3 \end{pmatrix} \cdot \begin{pmatrix} 3 \\ -4 \\ 1 \end{pmatrix} = 12 + 4 + 3 = \sqrt{4^2 + (-1)^2 + 3^2} \sqrt{3^2 + (-4)^2 + 1^2} \cos \theta$	M1 A1
	$\cos \theta = \frac{19}{26}$ awrt 0.73	A1 (3)
	(c) $X: (10, 0, 11)$ Accept vector forms	B1 (1)
	(d) $\vec{AX} = \begin{pmatrix} 10 \\ 0 \\ 11 \end{pmatrix} - \begin{pmatrix} -6 \\ 4 \\ -1 \end{pmatrix}$ Either order	M1
	$= \begin{pmatrix} 16 \\ -4 \\ 12 \end{pmatrix}$ cao	A1 (2)
	(e) $ \vec{AX} = \sqrt{16^2 + (-4)^2 + 12^2}$	M1
	$= \sqrt{416} = \sqrt{16 \times 26} = 4\sqrt{26} *$ Do not penalise if consistent incorrect signs in (d)	A1 (2)
	(f) 	Use of correct right angled triangle M1
	$\frac{ \vec{AX} }{d} = \cos \theta$ M1	
	$d = \frac{4\sqrt{26}}{\frac{19}{26}} \approx 27.9$ awrt 27.9	A1 (3)
[12]		

<p>5. (a)</p>	<p>Lines meet where: $\begin{pmatrix} -9 \\ 0 \\ 10 \end{pmatrix} + \lambda \begin{pmatrix} 2 \\ 1 \\ -1 \end{pmatrix} = \begin{pmatrix} 3 \\ 1 \\ 17 \end{pmatrix} + \mu \begin{pmatrix} 3 \\ -1 \\ 5 \end{pmatrix}$</p> <p>i: $-9 + 2\lambda = 3 + 3\mu$ (1)</p> <p>Any two of j: $\lambda = 1 + \mu$ (2)</p> <p>k: $10 - \lambda = 17 + 5\mu$ (3)</p> <p>(1) – 2(2) gives: $-9 = 1 + 5\mu \Rightarrow \mu = -2$ (M1)</p> <p>(2) gives: $\lambda = 1 - 2 = -1$ (A1)</p> <p>$\mathbf{r} = \begin{pmatrix} -9 \\ 0 \\ 10 \end{pmatrix} + (-1) \begin{pmatrix} 2 \\ 1 \\ -1 \end{pmatrix}$ or $\mathbf{r} = \begin{pmatrix} 3 \\ 1 \\ 17 \end{pmatrix} + (-2) \begin{pmatrix} 3 \\ -1 \\ 5 \end{pmatrix}$ (M1)</p> <p>Intersect at $\mathbf{r} = \begin{pmatrix} -3 \\ 3 \\ 7 \end{pmatrix}$ or $\mathbf{r} = -3\mathbf{i} + 3\mathbf{j} + 7\mathbf{k}$ (A1)</p> <p>Either check k:</p> <p>$\lambda = -1$: LHS = $10 - \lambda = 10 - (-1) = 11$</p> <p>$\mu = -2$: RHS = $17 + 5\mu = 17 - 10 = 7$</p> <p>(b) $\mathbf{d}_1 = 2\mathbf{i} + \mathbf{j} - \mathbf{k}$, $\mathbf{d}_2 = 3\mathbf{i} - \mathbf{j} + 5\mathbf{k}$</p> <p>As $\mathbf{d}_1 \cdot \mathbf{d}_2 = \begin{pmatrix} 2 \\ 1 \\ -1 \end{pmatrix} \cdot \begin{pmatrix} 3 \\ -1 \\ 5 \end{pmatrix} = (2 \times 3) + (1 \times -1) + (-1 \times 5) = 0$ (M1 A1 (2))</p> <p>Then l_1 is perpendicular to l_2.</p> <p>(c) Equating i; $-9 + 2\lambda = 5 \Rightarrow \lambda = 7$</p> <p>$\mathbf{r} = \begin{pmatrix} -9 \\ 0 \\ 10 \end{pmatrix} + 7 \begin{pmatrix} 2 \\ 1 \\ -1 \end{pmatrix} = \begin{pmatrix} 5 \\ 7 \\ 3 \end{pmatrix}$ ($= \overrightarrow{OA}$. Hence the point A lies on l_1.) (B1 (1))</p> <p>(d) Let $\overrightarrow{OX} = -3\mathbf{i} + 3\mathbf{j} + 7\mathbf{k}$ be point of intersection</p> <p>$\overrightarrow{AX} = \overrightarrow{OX} - \overrightarrow{OA} = \begin{pmatrix} -3 \\ 3 \\ 7 \end{pmatrix} - \begin{pmatrix} 5 \\ 7 \\ 3 \end{pmatrix} = \begin{pmatrix} -8 \\ -4 \\ 4 \end{pmatrix}$ (M1 ft)</p> <p>$\overrightarrow{OB} = \overrightarrow{OA} + \overrightarrow{AB} = \overrightarrow{OA} + 2\overrightarrow{AX}$</p> <p>$\overrightarrow{OB} = \begin{pmatrix} 5 \\ 7 \\ 3 \end{pmatrix} + 2 \begin{pmatrix} -8 \\ -4 \\ 4 \end{pmatrix}$ (M1 ft)</p> <p>Hence, $\overrightarrow{OB} = \begin{pmatrix} -11 \\ -1 \\ 11 \end{pmatrix}$ or $\overrightarrow{OB} = -11\mathbf{i} - \mathbf{j} + 11\mathbf{k}$ (A1)</p>	<p>(12 marks)</p>
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Question Number	Scheme	Marks
6. (a)	$\left\{ \begin{array}{l} u = x \Rightarrow \frac{du}{dx} = 1 \\ \frac{dv}{dx} = \cos 2x \Rightarrow v = \frac{1}{2} \sin 2x \end{array} \right\}$	
	<p>(see note below) Use of 'integration by parts' formula in the correct direction. Correct expression.</p> $\text{Int} = \int x \cos 2x \, dx = \frac{1}{2} x \sin 2x - \int \frac{1}{2} \sin 2x \cdot 1 \, dx$ $= \frac{1}{2} x \sin 2x - \frac{1}{2} \left(-\frac{1}{2} \cos 2x \right) + c$ $= \frac{1}{2} x \sin 2x + \frac{1}{4} \cos 2x + c$ <p>Correct expression with +c</p>	<p>M1 A1 dM1 A1</p> <p>[4]</p>
(b)	$\int x \cos^2 x \, dx = \int x \left(\frac{\cos 2x + 1}{2} \right) dx$ $= \frac{1}{2} \int x \cos 2x \, dx + \frac{1}{2} \int x \, dx$ $= \frac{1}{2} \left(\frac{1}{2} x \sin 2x + \frac{1}{4} \cos 2x \right) + \frac{1}{2} \int x \, dx$ $= \frac{1}{4} x \sin 2x + \frac{1}{8} \cos 2x + \frac{1}{4} x^2 (+c)$	
	<p>Substitutes correctly for $\cos^2 x$ in the given integral</p> <p>$\frac{1}{2}$ (their answer to (a)); or <u>underlined expression</u></p> <p>Completely correct expression with/without +c</p>	<p>M1 A1; $\sqrt{\quad}$ A1</p> <p>[3]</p>
		7 marks

7.	<p>(a) At $P(4, 2\sqrt{3})$ either $4 = 8\cos t$ or $2\sqrt{3} = 4\sin 2t$</p> <p>\Rightarrow only solution is $t = \frac{\pi}{3}$ where $0 \leq t \leq \frac{\pi}{2}$</p>	M1 A1
	<p>(b) $x = 8\cos t$, $y = 4\sin 2t$</p> <p>$\frac{dx}{dt} = -8\sin t$, $\frac{dy}{dt} = 8\cos 2t$</p> <p>At P, $\frac{dy}{dx} = \frac{8\cos(\frac{2\pi}{3})}{-8\sin(\frac{\pi}{3})}$</p> <p>$\left\{ = \frac{8(-\frac{1}{2})}{(-8)(\frac{\sqrt{3}}{2})} = \frac{1}{\sqrt{3}} = \text{awrt } 0.58 \right\}$</p> <p>Hence $m(N) = -\sqrt{3}$ or $-\frac{1}{\sqrt{3}}$</p> <p>N: $y - 2\sqrt{3} = -\sqrt{3}(x - 4)$</p> <p>N: $y = -\sqrt{3}x + 6\sqrt{3}$ (*)</p>	M1 A1 M1 M1 A1 cso (6)
	<p>(c) $A = \int_0^4 y dx = \int_{\frac{\pi}{2}}^{\frac{\pi}{3}} 4\sin 2t \cdot (-8\sin t) dt$</p> <p>$A = \int_{\frac{\pi}{2}}^{\frac{\pi}{3}} -32\sin 2t \cdot \sin t dt = \int_{\frac{\pi}{2}}^{\frac{\pi}{3}} -32(2\sin t \cos t) \cdot \sin t dt$</p> <p>$A = \int_{\frac{\pi}{2}}^{\frac{\pi}{3}} -64 \cdot \sin^2 t \cos t dt$</p> <p>$A = \int_{\frac{\pi}{3}}^{\frac{\pi}{2}} 64 \cdot \sin^2 t \cos t dt$ (*)</p>	M1 A1 M1 A1 (4)
	<p>(d) $A = 64 \left[\frac{\sin^3 t}{3} \right]_{\frac{\pi}{3}}^{\frac{\pi}{2}}$ or $A = 64 \left[\frac{u^3}{3} \right]_{\frac{\sqrt{3}}{2}}^1$</p> <p>$A = 64 \left[\frac{1}{3} - \left(\frac{1}{3} \cdot \frac{\sqrt{3}}{2} \cdot \frac{\sqrt{3}}{2} \cdot \frac{\sqrt{3}}{2} \right) \right]$</p> <p>$A = 64 \left(\frac{1}{3} - \frac{1}{8}\sqrt{3} \right) = \frac{64}{3} - 8\sqrt{3}$</p>	M1 A1 M1 A1 (4) (16 marks)

Question 1

Most candidates attempted this question and many achieved full marks.

In part (a), most candidates were able to differentiate implicitly to gain the first three marks. A minority of candidates struggled to apply the product rule correctly on $3x^2y$. At this point a minority of candidates substituted $x = -1$ and $y = 1$ in their differentiated equation, but the majority of candidates proceeded to find an expression for $\frac{dy}{dx}$ in terms of x and y , before substituting in these values. Although the majority of candidates were able to find the correct answer of $-\frac{4}{9}$, common errors in this part included sign errors either in rearranging or when substituting $x = -1$ and $y = 1$ into their $\frac{dy}{dx}$ expression. A small number of candidates tried to rearrange the equation given in the question in order to make y the subject. This resulted in very few, if any, marks being awarded.

In part (b), a small minority of candidates either found the equation of the tangent and gained no marks or did not give their equation of the normal in the form $ax + by + c = 0$, where a , b and c are integers, and lost the final accuracy mark.

Question 2

This question was a good starting question and over 60% of the candidates gained full marks. A few candidates used a wrong angle mode when calculating the values in part (a). In part (b), the majority knew the structure of the trapezium rule. The most common errors were to miscalculate the interval width using, for example, $\frac{\pi}{9}$ and $\frac{\pi}{15}$ in place of $\frac{\pi}{12}$ and $\frac{\pi}{24}$. Some were unable to adapt to the situation in which they did not need all the information given in the question to solve part of it and either used the same interval width for (b)(i) and (b)(ii) or answered b(ii) only. A few answered b(ii) only and proceeded to attempt to find an exact answer using analytic calculus, which in this case is impossible. These candidates were apparently answering the question that they expected to be set rather than the one which had actually been set. In Mathematics, as in all other subjects, carefully reading and answering the question as set are necessary examination skills.

Question 3

Nearly all candidates gained some marks in part (a) realising that they had to divide $\frac{dy}{dt}$ by $\frac{dx}{dt}$. Most could differentiate $\sqrt{3} \sin 2t$ correctly, although occasionally dividing by 2, instead of multiplying, was seen. Differentiating $4\cos^2 t$ proved more difficult. Many had to use a double angle formula and this led to many errors in signs and constants. $\frac{d}{dt}(4\cos^2 t) = k \sin^2 t$, where k might be ± 4 , ± 8 or ± 2 was also frequently seen. Many who

correctly obtained $\frac{dy}{dx} = -\frac{2}{\sqrt{3}} \tan 2t$ were unable to transform this correctly to the form specified in the question and, in this context, surd manipulation was a weak area. Nearly all knew how to solve part (b), a C1 topic, and, if they had a correct answer to part (a), gained full marks here.

Part (c) proved demanding and only about 15% of the candidates were able to complete the question correctly. Many realised that they had to use a double angle formula and gained the first mark, either by writing $x = 2\sqrt{3} \sin t \cos t$ or $y = \cos 2t + 2$. Although other approaches are possible, the most commonly successful method was to use $\sin^2 \theta + \cos^2 \theta = 1$, where θ is either t or $2t$, to eliminate the parameter. There are many alternative forms of the answer to this question.

Some otherwise correct answers, for example, $x = \sqrt{3} \left(1 - \frac{\sqrt{y}}{2} \right) \sqrt{y}$, lost the final mark as the answer only gave half of the curve.

Question 4

The majority of candidates made good attempts at parts (a) to (e) of this question. Many, however, wasted a good deal of time in part (a), proving correctly that $\lambda = \mu = 0$ before obtaining the correct answer. When a question starts “Write down ...”, then candidates should realise that no working is needed to obtain the answer. The majority of candidates knew how to use the scalar product to find the cosine of the angle and chose the correct directions for the lines. Parts (c) and (d) were well done. In part (e), as in Q1(b), the working needed to establish the printed result was often incomplete. In showing that the printed result is correct, it is insufficient to proceed from $\sqrt{416}$ to $4\sqrt{23}$ without stating $416 = 16 \times 26$ or $4^2 \times 26$. Drawing a sketch, which many candidates seem reluctant to do, shows that part (f) can be solved by simple trigonometry, using the results of parts (b) and (e). Many made no attempt at this part and the majority of those who did opted for a method using a zero scalar product. Even correctly carried out, this is very complicated ($\mu = \frac{104}{19}$) and it was impressive to see some fully correct solutions. Much valuable time, however, had been wasted.

Question 5

In part (a), most candidates were able to set up and solve the three equations correctly. Some candidates either did not realise that they needed to perform a check for consistency or performed this check incorrectly. A surprising number of candidates did not follow the instruction in the question to find the position vector of the point of intersection. A few candidates were unable to successfully negotiate the absence of the \mathbf{j} term in $(-9\mathbf{i} + 10\mathbf{k})$ for l_1 and so formed incorrect simultaneous equations.

In part (b), a majority of candidates realised that they needed to apply the dot product formula on the direction vectors of l_1 and l_2 . Many of these candidates performed a correct dot product calculation but not all of them wrote a conclusion.

In part (c), a majority of candidates were able to prove that A lies on l_1 , either by substituting $\lambda = 7$ into l_1 or by checking that substituting $(5, 7, 3)$ into l_1 gave $\lambda = 7$ for all three components.

There was a failure by many candidates to see the link between part (d) and the other three parts of this question with the majority of them leaving this part blank. The most common error of those who attempted this part was to write down B as $-5\mathbf{i} - 7\mathbf{j} - 3\mathbf{k}$. Those candidates who decided to draw a diagram usually increased their chance of success. Most candidates who were successful at this part applied a vector approach as detailed in the mark scheme. The easiest vector approach, adopted by a few candidates, is to realise that $\lambda = 7$ at A , $\lambda = 3$ at the point of intersection and so $\lambda = -1$ at B . So substitution of $\lambda = -1$ into l_1 yields the correct position vector $-11\mathbf{i} - \mathbf{j} + 11\mathbf{k}$. A few candidates, by deducing that the intersection point is the midpoint of A and B were able to write down $\frac{x+5}{2} = -3$,

$\frac{y+7}{2} = 3$ and $\frac{z+3}{2} = 7$, in order to find the position vector of B .

Question 6

In part (a), many candidates recognised that the correct way to integrate $x \cos 2x$ was to use integration by parts, and many correct solutions were seen. Common errors included the incorrect integration of $\cos 2x$ and $\sin 2x$; the incorrect application of the ‘by parts’ formula even when the candidate had quoted the correct formula as part of their solution; and applying the by parts formula in the wrong direction by assigning $\frac{dy}{dx}$ as x to be integrated.

In part (b), fewer than half of the candidates deduced the connection with part (a) and proceeded by using “Way 1” as detailed in the mark scheme. A significant number of candidates used integration by parts on $\int x \left(\frac{\cos 2x + 1}{2} \right) dx$ and proceeded by using “Way 2” as detailed in the mark scheme.

In part (b), the biggest source of error was in the rearranging and substituting of the identity into the given integral. Some candidates incorrectly rearranged the $\cos 2x$ identity to give $\cos^2 x = \frac{\cos 2x - 1}{2}$. Other candidates used brackets incorrectly and wrote $\int x \cos^2 x dx$ as either $\int \left(\frac{x}{2} \cos 2x + 1 \right) dx$ or $\int \left(\frac{x}{2} \cos 2x + \frac{1}{2} \right) dx$.

A significant number of candidates omitted the constant of integration in their answers to parts (a) and (b). Such candidates were penalised once for this omission in part (a).

Question 8

In part (a), many candidates were able to give $t = \frac{\pi}{3}$. Some candidates gave their answer only in degrees instead of radians. Other candidates substituted the y -value of P into $y = 4 \sin 2t$ and found two values for t , namely $t = \frac{\pi}{6}, \frac{\pi}{3}$. The majority of these candidates did not go on to reject $t = \frac{\pi}{6}$. In part (b), many candidates were able to apply the correct formula for finding $\frac{dy}{dx}$ in terms of t , although some candidates erroneously believed that differentiation of $4 \sin 2t$ gave either $-8 \cos 2t$, $8 \cos t$ or $2 \cos 2t$. Some candidates who had differentiated incorrectly, substituted their value of t into $\frac{dy}{dx}$ and tried to “fudge” their answer for $\frac{dy}{dx}$ as $\frac{1}{\sqrt{3}}$, after

realising from the given answer that the gradient of the normal was $-\sqrt{3}$. The majority of candidates understood the relationship between the gradient of the tangent and its normal and many were able to produce a fully correct solution to this part.

A few candidates, however, did not realise that parametric differentiation was required in part (b) and some of these candidates tried to convert the parametric equations into a Cartesian equation. Although some candidates then went on to attempt to differentiate their Cartesian equation, this method was rarely executed correctly.

Few convincing proofs were seen in part (c) with a significant number of candidates who were not aware with the procedure of reversing the limits and the sign of the integral. Therefore, some candidates conveniently differentiated $8\cos t$ to give “positive” $8\sin t$, even though they had previously differentiated $8\cos t$ correctly in part (a). After completing this part, some candidates had a ‘crisis of confidence’ with their differentiation rules and then went on to amend their correct solution to part (a) to produce an incorrect solution. Other candidates differentiated $8\cos t$ correctly but wrote their limits the wrong way round giving

$A = \int_{\frac{\pi}{3}}^{\frac{\pi}{2}} 4\sin 2t \cdot (-8\sin t) dt$ and after stating $\sin 2t = 2\cos t \sin t$ (as many candidates were able to do in this part) wrote $A = \int_{\frac{\pi}{3}}^{\frac{\pi}{2}} -64\sin^2 t \cos t dt$. These candidates then wrote down the given answer by arguing that all areas should be positive.

Part (d) was unstructured in the sense that the question did not tell the candidates how to integrate the given expression. Only a minority of candidates spotted that a substitution was required, although some candidates were able to integrate $64\sin^2 t \cos t$ by inspection. Many candidates replaced $\sin^2 t$ with $\frac{1}{2}(1 - \cos 2t)$ but then multiplied this out with $\cos t$ to give $\frac{1}{2}\cos t - \frac{1}{2}\cos t \cos 2t$. Very few candidates correctly applied the sum-product formula on this expression, but most candidates usually gave up at this point or went on to produce some incorrect integration. Other candidates replaced $\sin^2 t$ with $1 - \cos^2 t$, but did not make much progress with this. A significant number of candidates used integration by parts with a surprising number of them persevering with this technique more than once before deciding they could make no progress. It is possible, however, to use a ‘loop’ method but this was very rarely seen. It was clear to examiners that a significant number of stronger candidates spent much time trying to unsuccessfully answer this part with a few candidates producing at least two pages of working.

Statistics for C4 Practice Paper Silver Level S5

Qu	Max score	Modal score	Mean %	Mean score for students achieving grade:							
				ALL	A*	A	B	C	D	E	U
1	8		85	6.76	7.85	7.44	6.93	6.36	5.46	4.68	1.82
2	8		82	6.53	7.82	7.40	6.87	6.17	5.25	4.15	2.35
3	12		62	7.45	11.14	9.47	7.67	5.80	4.01	2.40	1.00
4	12		59	7.02		9.02	6.07	4.47	3.15	2.00	0.77
5	12		57	6.81		9.07	7.04	5.25	3.60	2.26	0.94
6	7		54	3.76		5.57	3.77	2.40	1.30	0.65	0.25
7	16		48	7.73		11.16	7.39	5.28	3.41	2.15	1.08
	75		61	46.06		59.13	45.74	35.73	26.18	18.29	8.21